

Chapter 3 – More Compound Interest Topics

In this chapter, we will extend our knowledge of compound interest to include solving problems where N and i are the unknown variables. While problems of this sort are not as common as problems requiring the solution of the PV or FV, we still need to know how to work them. In this chapter, we will again use this formula:

$$FV = PV \times (1 + i)^N \quad (\text{Formula 2-1})$$

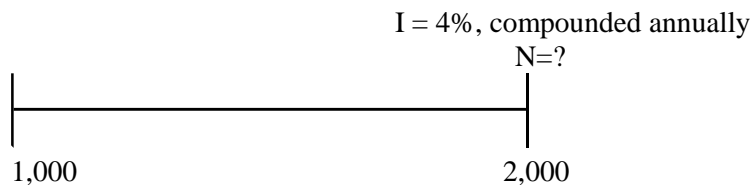
This chapter assumes you have worked through the appendix to Chapter 2.

Section 1 – Solving for N

Many problems arise where we know all the information about a situation except for the value of N . Consider this first example.

Example 1. Jack has deposited \$1,000 into a savings account. He wants to withdraw it when it has grown to \$2,000. If the interest rate is 4% compounded annually, how long will he have to wait?

Our timeline would look like this:



When we place these values into the compound interest formula it looks like this:

$$2,000 = 1,000 \times (1 + 0.04)^N$$

Unfortunately, there is no simple way from algebra to rearrange this formula to say $N = \dots$ without using a tool called logarithms.

Logarithms are an old tool used to simplify working with exponents or powers. There are several “Laws” of Logarithms that define what applying a logarithm will do to an equation (Note: we will call logarithms by the shorthand “log” rather than spelling out the entire name from now on). In our situation, we need to get that exponent “ N ” down on the primary equation line so that we can try to rearrange it to the “ $N = \dots$ ” form. We can't divide both sides by N , that won't work. Neither can we divide both sides by the $1000 \times (1 + 0.04)$ either. Logs come to the rescue. First, let's rearrange the formula to look like this, by dividing both sides by 1,000. Now we have it in the form we need to use logs to help us. But, first, another mathematical diversion.

$$\frac{2,000}{1,000} = (1 + 0.04)^N$$

Chapter 3 – More Compound Interest Topics

Mathematical Diversion #2 – Using logarithms

If you have an equation of the form $8 = 2^N$, we can solve this by taking the log of both sides of the equation. This gives us an equation that looks like this: $\text{LOG}(8) = \text{LOG}(2^N)$. Great? What does that do for us, you say? Nothing yet. However, one of those “Laws” of logs states that the log of a number to a power is equal to the power multiplied by the log of that number. So, we can rearrange this equation again like this: $\text{LOG}(8) = N \times \text{LOG}(2)$. We can now rearrange this equation and solve it by taking the log of 8 divided by the log of 2. The value of N for which 2 taken to that power is equal to 8 is the value of 3.

The key on the HP-12C that calculates logs is the **LN** key, which is accessed by pressing the **blue shift** key and then the **%T** key to access the **LN** function written in blue on that key. On the TI BAI PLUS, the **LN** function is on top of a key right above the **STO** key on the left side. On the HP-10B, the **LN** function is written in yellow above the **2** key. It is accessed by pressing the **yellow shift** key and then the **2** key. These keys are labeled **LN** which stands for “Log, Natural”, (compared to LOG which stands for common logarithms), a distinction that won't matter to us.

If we take the log of both sides of our formula, it now looks like this:

$$\text{LOG}\left(\frac{2,000}{1,000}\right) = N \times \text{LOG}(1 + 0.04)$$

We can solve for N by dividing to get N by itself. The equation now looks like this:

$$N = \frac{\text{LOG}\left(\frac{2,000}{1,000}\right)}{\text{LOG}(1 + 0.04)}$$

To solve this equation, press these keys:

HP-10B
2000 ÷ 1000 =
yellow shift LN ÷ (1 +
0.04) yellow shift LN =

HP-12C
2000 ENTER 1000 ÷
blue shift LN 1 ENTER 0.04
+ blue shift LN ÷

TI BAI Plus
2000 ÷ 1000 = LN ÷ (1 +
0.04) LN =

The answer of 17.67 is shown in the display.

However, solving this problem is much easier if we use the built-in business keys. Compare these solutions to the one we just went through.

HP-10B
4 I/YR
1000 PV
2000 +/- FV
0 PMT
N

HP-12C
4 i
1000 PV
2000 CHS FV
0 PMT
n

TI BAI Plus
4 I/Y
1000 PV
2000 +/- FV
0 PMT
CPT N

And the answer of 17.67 is shown in the display. Note that the HP-12C returns the answer of 18. For some reason, Hewlett Packard made the built-in routine that solves for N round up to the nearest whole number.

Chapter 3 – More Compound Interest Topics

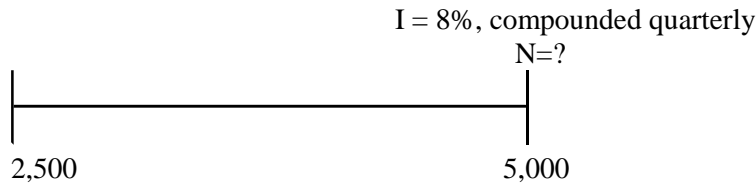
On the HP-12C, we can approximate the correct value for N by doing this:

When 18 is shown, press **17** and **n**. Now press **FV**. Note that the value displayed is $-\$1,947.90$. Store this in memory 0 by pressing **STO 0**. Enter **2000** and press **+**. You should see 52.10 in the display. Store this in memory 1 by pressing **STO 1**. Now enter 18 and press **n**. Press **FV** again. You should see $-\$2,025.82$ in the display. Press **RCL 0**, **CHS**, and **+**. You should now see -77.92 in the display. Press **RCL 1** and then press the **X<>Y** key, **CHS**, and then **+**. If we add this value to the 17 we used originally, we have 17.66, which is very close to the actual value of 17.67. Close enough, in fact, that the difference probably won't matter to anyone!

Using the financial keys on our business calculators is so MUCH easier than using logs. So much so that we will not use logs again.

Example 2. How many years will it take for a \$2,500 deposit to double at an interest rate of 8%, compounded quarterly?

The timeline looks like this:



Notice that we have already written down 5,000 over on the FV end of the timeline. Although the problem didn't specifically mention \$5,000, that is what doubling \$2,500 will give.

The formula looks like this:

$$5,000 = 2,500 \times \left(1 + \frac{0.08}{4}\right)^N$$

To solve this problem, press these keys:

HP-10B
8 ÷ 4 = I/YR
2500 PV
5000 +/- FV
0 PMT
N

HP-12C
8 ENTER 4 ÷ i
2500 PV
5000 CHS FV
0 PMT
n

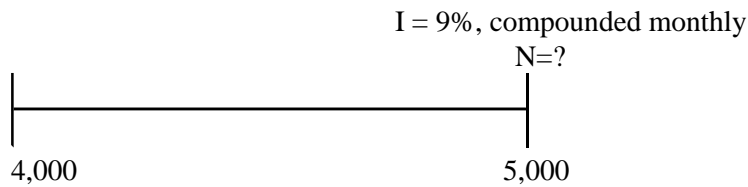
TI BAI Plus
8 ÷ 4 = I/Y
2500 PV
5000 +/- FV
0 PMT
CPT N

And the answer of 35.00 is shown in the display. This is the number of quarters. Divide by 4 to convert to years. The answer is 8.75 years. The HP-12C shows an answer of 35.00 as well. This is because the answer appears to be 35.00 to 2 digits. It is actually 35.002789. Try this problem again on the HP-12C and enter the FV as $-5,001$. This time, the HP-12C returns the value of 36 for N! What's the moral of this story? Be very careful when solving for N with the HP-12C.

Example 3. How many months will it take for a \$4,000 deposit to grow to \$5,000 at an interest rate of 9%, compounded monthly?

Chapter 3 – More Compound Interest Topics

The timeline and formula look like this:



$$5,000 = 4,000 \times \left(1 + \frac{0.09}{12}\right)^N$$

To solve this problem, press these keys:

HP-10B
9 ÷ 12 = I/YR
4000 PV
5000 +/- FV
0 PMT
N

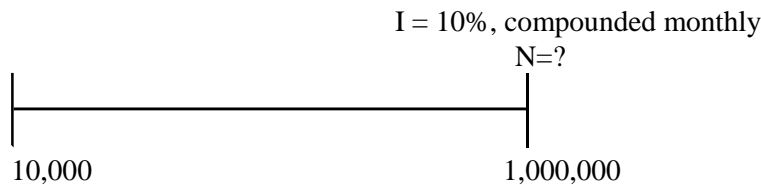
HP-12C
9 ENTER 12 ÷ i
4000 PV
5000 CHS FV
0 PMT
n

TI BAI Plus
9 ÷ 12 = I/Y
4000 PV
5000 +/- FV
0 PMT
CPT N

And the answer of 29.86 months is shown in the display. The HP-12C shows an answer of 30 months in the display.

Example 4. Jack has just received a \$10,000 inheritance. He decides to deposit it into an account paying 10%, compounded monthly. How long must he leave the money on deposit before he has a balance of \$1,000,000?

The timeline and formula look like this:



$$1,000,000 = 10,000 \times \left(1 + \frac{0.10}{12}\right)^N$$

To solve this problem, press these keys:

HP-10B
10 ÷ 12 = I/YR
10000 PV
1000000 +/- FV
0 PMT
N

HP-12C
10 ENTER 12 ÷ i
10000 PV
1000000 CHS FV
0 PMT
n

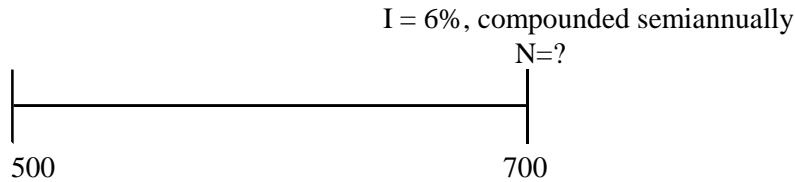
TI BAI Plus
10 ÷ 12 = I/Y
10000 PV
1000000 +/- FV
0 PMT
CPT N

And the answer of 554.92 months is shown in the display. How long is that? 46.24 years! The HP-12C shows an answer of 555 months in the display.

Chapter 3 – More Compound Interest Topics

Example 5. How many semiannual periods would you need to leave a deposit of \$500 in an account paying 6% compounded semiannually before it would accumulate to \$700?

The timeline looks like this:



The formula looks like this:

$$700 = 500 \times \left(1 + \frac{0.06}{2}\right)^N$$

To solve this problem, press these keys:

HP-10B
6 ÷ 2 = I/YR
500 PV
700 +/- FV
0 PMT
N

HP-12C
6 ENTER 2 ÷ i
500 PV
700 CHS FV
0 PMT
n

TI BAII Plus
6 ÷ 2 = I/Y
500 PV
700 +/- FV
0 PMT
CPT N

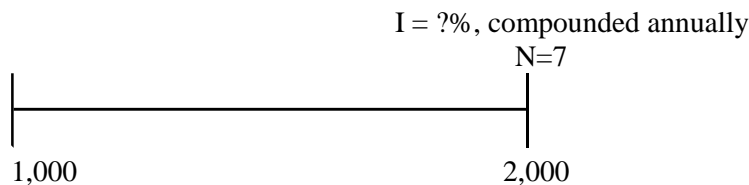
And the answer of 11.38 semiannual periods is shown in the display. That's 5.69 years! The HP-12C shows an answer of 12 semiannual periods in the display.

Section 2 – Solving for i

The other part of the compound interest formula that we have not yet solved for is the interest rate, i . Problems requiring us to solve for the interest rate are not as prevalent as some others, but solving for this unknown is not that difficult. Let's try an example.

Example 6. What interest rate, compounded annually, would cause an initial \$1,000 deposit to grow to \$2,000 in 7 years?

The timeline looks like this:



$$2,000 = 1,000 \times (1 + i)^7$$

To solve this problem, we'll need to rearrange the formula somewhat to look like this:

Chapter 3 – More Compound Interest Topics

$$\frac{2,000}{1,000} = (1+i)^7$$

To get beyond this point, however, we'll need another mathematical diversion to get i by itself.

Mathematical Diversion #3 – Finding roots

Suppose you had an equation of the form $4 = X^2$. How would you solve it? Probably you'd suggest taking the square root of both sides. Then you would have this:

$$\sqrt{4} = \sqrt{X^2}$$

However, the square root of X^2 is just X . So what you really have is just

$$\sqrt{4} = X$$

and you could solve that by entering 4 on your calculator and pushing the square root button.

Our situation is helped if we realize that a square root is actually taking a number to the $1/2$ power. This means that you could find the square root of any number by using the Y^X key on your calculator – the square root button is actually unnecessary!

Suppose you had an equation like this: $8 = X^3$. You could actually take the “third” root of each side of this equation or just take both sides of the equation to the $1/3$ power (which is the same thing). This means that you could solve this equation by performing these keystrokes on the HP-10B: **8 yellow shift Y^X 3 yellow shift $1/X$ =** and you'd see the answer of 2. On the TI BAI PLUS, press these keys: **8 Y^X 3 $1/X$ =**. On the HP-12C, these keystrokes would work: **8 ENTER 3 $1/X$ Y^X** . Simple!

In our example, we need to take the “seventh” root of both sides of the equation (or take both sides of the equation to the $1/7^{\text{th}}$ power).

This changes our formula to look like this:

$$\sqrt[7]{\frac{2,000}{1,000}} = \sqrt[7]{(1+i)^7}$$

Which reduces to this:

$$\sqrt[7]{\frac{2,000}{1,000}} = (1+i)$$

since the 7^{th} root of something raised to the 7^{th} power would cancel each other out! We can now rewrite the formula in a way where we can show i = as follows by rearranging it again. We can solve this form!

$$i = \left(\sqrt[7]{\frac{2,000}{1,000}} - 1 \right)$$

Chapter 3 – More Compound Interest Topics

We solve this formula by pressing these keys:

HP-10B

2000 ÷ 1000 =

yellow shift Y^x 7

yellow shift 1/X = - 1 =

HP-12C

2000 ENTER 1000 ÷ 7 1/X

Y^x 1 -

TI BAII Plus

2000 ÷ 1000 = Y^x 7 1/X =

- 1 =

The answer is 0.1041 to four decimal places. That's 10.41% for the interest rate.

However, we can simplify the solution to problems solving for i by using the business keys on our business calculators just like we did for problems solving for N .

To solve this problem, press these keys:

HP-10B

7 N

1000 PV

2000 +/- FV

0 PMT

I/YR

HP-12C

7 n

1000 PV

2000 CHS FV

0 PMT

i

TI BAII Plus

7 N

1000 PV

2000 +/- FV

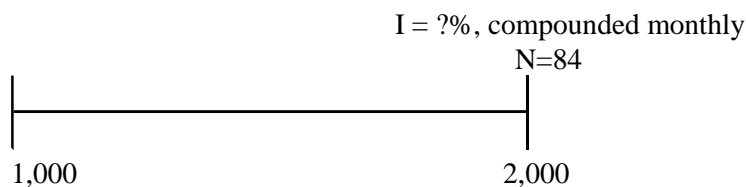
0 PMT

CPT I/Y

And the answer of 10.41 is shown in the display. That makes these problems as easy to solve as any other business math problems!

Example 7. What interest rate, compounded monthly, would cause an initial \$1,000 deposit to grow to \$2,000 in 7 years?

The timeline looks like this:



Note that there are 84 months in 7 years. The formula looks like this:

$$2,000 = 1,000 \times \left(1 + \frac{i}{12}\right)^{84}$$

To solve this problem, press these keys:

HP-10B

84 N

1000 PV

2000 +/- FV

0 PMT

I/YR

HP-12C

84 n

1000 PV

2000 CHS FV

0 PMT

i

TI BAII Plus

84 N

1000 PV

2000 +/- FV

0 PMT

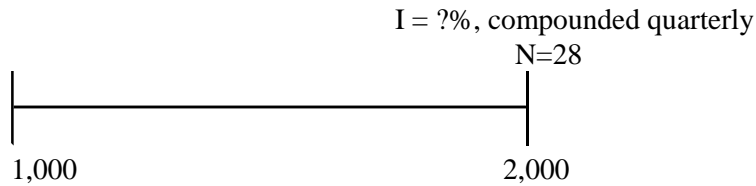
CPT I/Y

The answer of 0.8286 is shown in the display. However, this is the interest rate per month. We should change this into an interest rate compounded monthly, since that is how we normally view interest rates. To do that, multiply this rate by 12. The answer is 9.94%, compounded monthly.

Chapter 3 – More Compound Interest Topics

Example 8. What interest rate, compounded quarterly, would cause an initial \$1,000 deposit to grow to \$2,000 in 7 years?

The timeline looks like this:



Note that there are 28 quarters in 7 years. The formula looks like this:

$$2,000 = 1,000 \times \left(1 + \frac{i}{4}\right)^{28}$$

To solve this problem, press these keys:

HP-10B
28 N
1000 PV
2000 +/- FV
0 PMT
I/YR

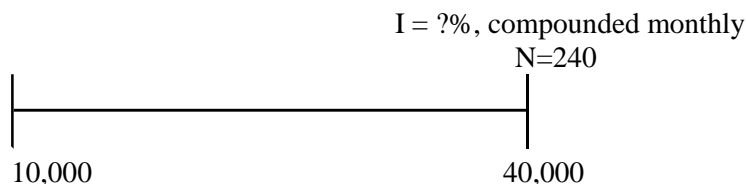
HP-12C
28 n
1000 PV
2000 CHS FV
0 PMT
i

TI BAII Plus
28 N
1000 PV
2000 +/- FV
0 PMT
CPT I/Y

The answer of 2.5064 is shown in the display. When multiplied by 4 (since the 2.5064 number is the interest rate per period), the answer is 10.03%, compounded quarterly.

Example 9. What interest rate, compounded monthly, would cause an initial \$10,000 deposit to grow to \$40,000 in 20 years?

The timeline and formula look like this: Note that there are 240 months in 20 years.



$$40,000 = 10,000 \times \left(1 + \frac{i}{12}\right)^{240}$$

To solve this problem, press these keys:

HP-10B
240 N
10000 PV
40000 +/- FV
0 PMT
I/YR

HP-12C
240 n
10000 PV
40000 CHS FV
0 PMT
i

TI BAII Plus
240 N
10000 PV
40000 +/- FV
0 PMT
CPT I/Y